Recent developments in the modeling of heavy quarkonia

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Outline

1. Introduction.
2. Overview of potential model approaches.
3. Results
   3.a Charmonium spectrum and decays
   3.b Upsilon spectrum and decays
4. Conclusions and Outlook.

1. Introduction

Over the past 25+ years, potential models have proven valuable in analyzing the spectra and characteristics of heavy quarkonium systems. Motivation for revisiting the potential model interpretation of the $c\bar{c}$ and $b\bar{b}$ systems is provided by recent experimental results:

- The discovery of several expected states in the charmonium spectrum (η'_c and h_c);
- The discovery of a state [X(3872)], which could be a $^3D_2$ charmonium level;
- The discovery of the 1^3D_2 state of the upsilon system;
- The discovery of a $b\bar{c}$ state (B_c);
- The determination of various decay widths (e^+e^-, E_1);
- Etc.

For potential models, the questions with regard to the charmonium and upsilon systems seem to be:

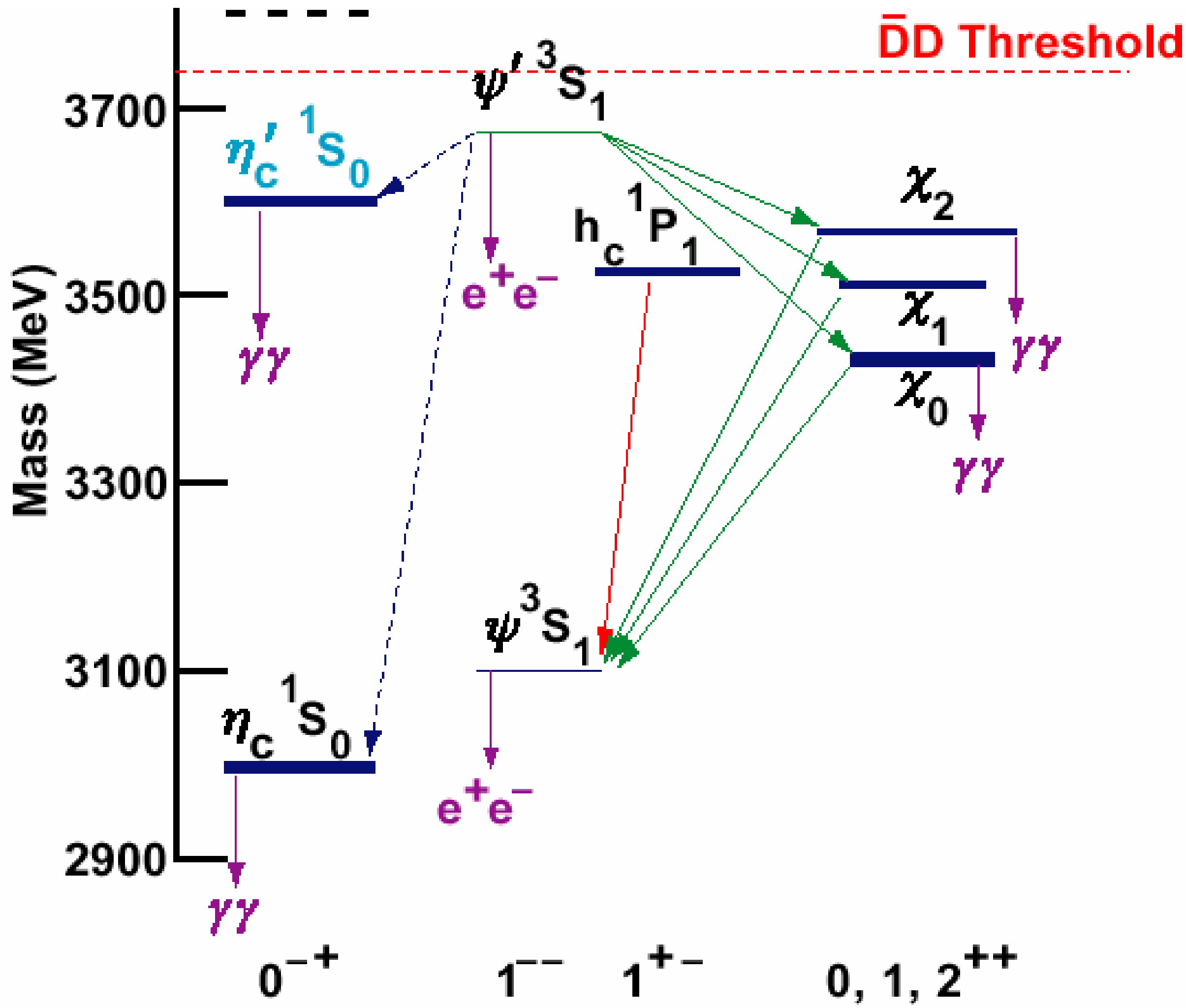
- Can potential models describe the spin splittings in a quantitatively satisfactory way?
- All potential models contain a phenomenological confining potential. What are its Lorentz properties?
- How well are the leptonic and radiative decays predicted?
- Can the newly discovered states [i.e., X(3872), X(3943)] be interpreted as fitting into quarkonium spectra?

Here, we will attempt to answer these questions using a potential model which includes the v^2/c^2 and all one-loop corrections to the short distance potential supplemented with a linear phenomenological confining potential and its v^2/c^2 corrections.

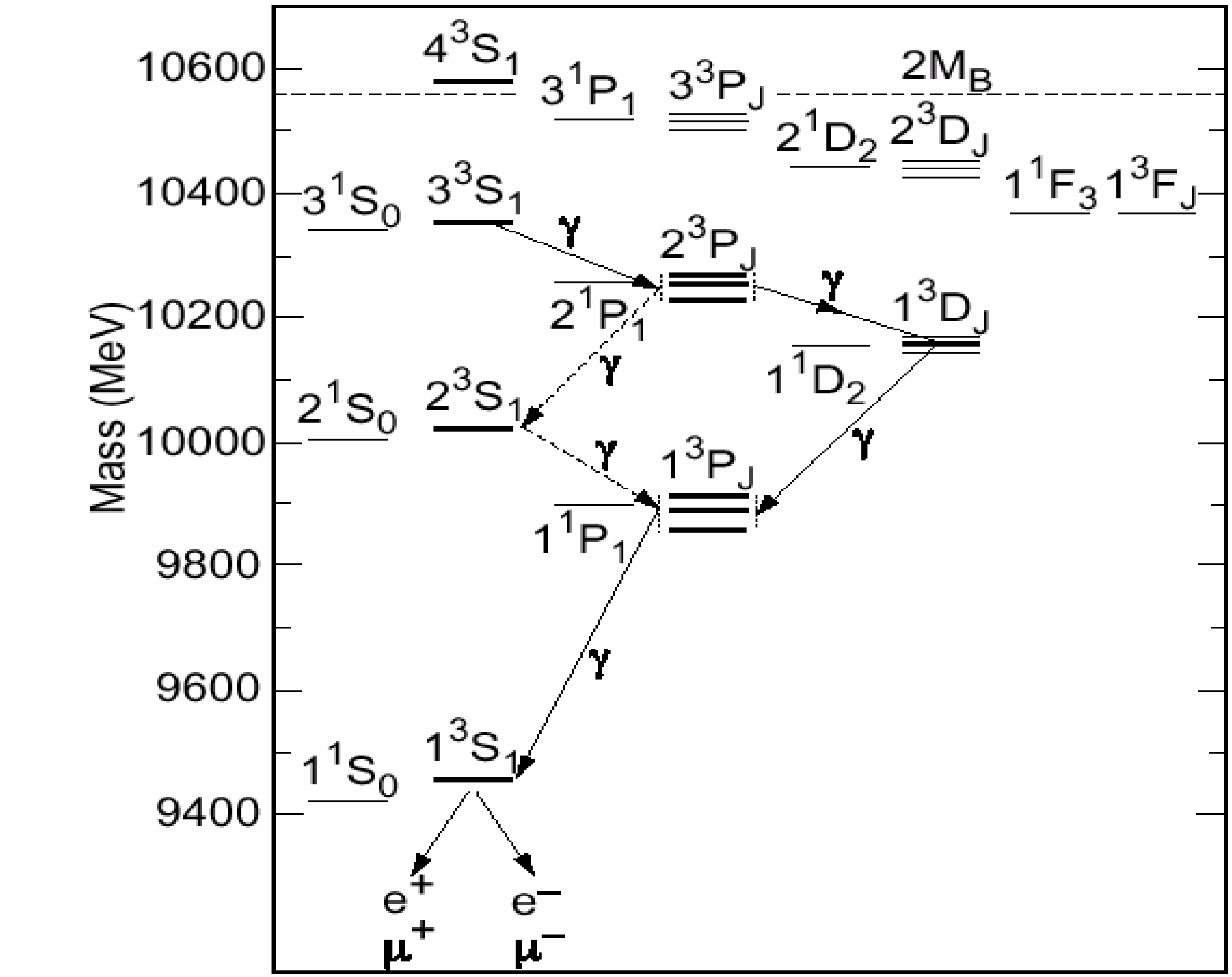
2. Overview of potential model approaches.

Early spin-independent models (Eichten, et al., 1974) were able to explain the nature of the J/Ψ ($c\bar{c}$) and its spin averaged spectrum. Inclusion of the Lorentz correction spin effects to this potential (Pumplin, WWR, Sato; Schnitzer, 1975) led to the prediction of a much richer spectral picture. Finally, by implementing a model which included the full one-loop QCD potential, we were able to adequately model the charmonium system and to predict the spectrum of the upsilon system with remarkable accuracy (Gupta, SFR, WWR, 1982). Other phenomenologically motivated potential models have been effectively employed, particularly for the prediction of decay widths of various states.

The known charmonium spectrum is shown below



The known and expected $b\bar{b}$ spectrum is shown here



From Gupta, Radford, Repko, 1982

TABLE II. $b\bar{b}$ spectrum with $m_b = 4.78$ GeV, $\mu = 3.75$ GeV, $\alpha_s(\mu) = 0.288$, and $A = 0.177$ GeV^2.

State	Mass (GeV)	State	Mass (GeV)
$1\,^3S_1(\Upsilon)$	9.462	$1\,^3D_3$	10.167
$1\,^1S_0(\eta_b)$	9.427	$1\,^3D_2$	$\boxed{10.162}$
		$1\,^3D_1$	10.155
$2\,^3S_1(\Upsilon')$	10.013	$1\,^1D_2$	10.163
$2\,^1S_0(\eta_b')$	9.994		
		$2\,^3D_3$	10.459
$3\,^3S_1(\Upsilon'')$	10.355	$2\,^3D_2$	10.454
$3\,^1S_0(\eta_b'')$	10.339	$2\,^3D_1$	10.447
		$2\,^1D_2$	10.455
$1\,^3P_2$	9.910		
$1\,^3P_1$	9.893	$1\,^3F_4$	10.365
$1\,^3P_0$	9.868	$1\,^3F_3$	10.364
$1\,^1P_1$	9.900	$1\,^3F_2$	10.361
		$1\,^1F_3$	10.364
$2\,^3P_2$	10.266		
$2\,^3P_1$	10.252		
$2\,^3P_0$	10.232		
$2\,^1P_1$	10.258		

$M(\Upsilon(^3D_2)) = 10161.1 \pm 0.6 \pm 1.6$ MeV (CLEO 2003)

Predictions for spin averaged levels can be obtained from a simple Hamiltonian of the form

$$H = \frac{\vec{p}^{\,2}}{2\mu} + V(r),$$

where the most notable choice for V(r) is the Cornell potential

$$V(r) = Ar - \frac{4}{3}\frac{\alpha_S}{r}.$$

The use of this simple potential, with the inclusion of continuum effects, has been remarkably successful in efforts to identify charmonium states which could be accessible to experiment. (Eichten, Lane and Quigg)

Spin effects can be included to order v^2/c^2 in a straightforward way (Pumplin, WWR & Sato, Schnitzer) to obtain a Hamiltonian of the form

$$H = \frac{\vec{p}^{\,2}}{2\mu} + V(r) + V_{HF} + V_{LS} + V_{TEN} + V_{SI}$$

where V_{SI} consists of spin-independent terms including the kinetic energy correction. For scalar + vector confinement, the confining potential is

$$V_L = (1 - f_V)V_S + f_V V_L$$

where f_V is the fraction of vector confinement and

$$V_S = Ar - \frac{A}{2m^2 r}\vec{L}\cdot\vec{S}$$

$$V_L = Ar + \frac{4A}{3m^2 r}\vec{S}_1\cdot\vec{S}_2 + \frac{3A}{2\,m^2 r}\vec{L}\cdot\vec{S} + \frac{A}{3m^2 r}(3\vec{S}_1\cdot\hat{r}\,\vec{S}_2\cdot\hat{r} - \vec{S}_1\cdot\vec{S}_2) + \frac{A}{2\,m^2 r}$$

To proceed beyond this level requires the inclusion of the one loop QCD corrections to the short distance potential (Gupta & SFR, 1981; Gupta, SFR & WWR).

$$V_{HF} = \frac{32\pi\alpha_S}{9m^2}\vec{S}_1\cdot\vec{S}_2\left[1-\frac{\alpha_S}{12\pi}\left(26+9\ln 2\right)\right]\delta(\vec{r})$$

$$+\frac{32\pi\alpha_S}{9m^2}\vec{S}_1\cdot\vec{S}_2\left\{-\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln(\mu r)+\gamma_E}{r}\right]+\frac{21\alpha_S}{16\pi^2}\nabla^2\left[\frac{\ln(mr)+\gamma_E}{r}\right]\right\}$$

$$V_{LS} = \frac{2\alpha_S}{m^2}\frac{\vec{L}\cdot\vec{S}}{r^3}\left\{1-\frac{11\alpha_S}{18\pi}+\frac{\alpha_S}{6\pi}(33-2n_f)(\ln\mu r+\gamma_E-1)-\frac{2\alpha_S}{\pi}(\ln mr+\gamma_E-1)\right\}$$

$$V_T = \frac{4\alpha_S}{3m^2}\frac{(3\vec{S}_1\cdot\hat{r}\,\vec{S}_2\cdot\hat{r}-\vec{S}_1\cdot\vec{S}_2)}{r^3}$$

$$\times\left\{1+\frac{4\alpha_S}{3\pi}+\frac{\alpha_S}{6\pi}(33-3n_f)(\ln\mu r+\gamma_E-\frac{4}{3})-\frac{3\alpha_S}{\pi}(\ln mr+\gamma_E-\frac{4}{3})\right\}$$

$$V_{SI} = \frac{4\pi\alpha_S}{3m^2}\left\{\left[1-\frac{\alpha_S}{2\pi}(1+\ln 2)\right]\delta(\vec{r})-\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln\mu r+\gamma_E}{r}\right]-\frac{7\alpha_S}{6\pi}\frac{m}{r^2}\right\}$$

To obtain the eigenvalues and wavefunctions for these complicated potentials it is convenient to use a variational approach. Specifically, we use trial wave functions of the form

$$\psi_{j\ell}^{m_j}(\vec{r}) = \sum_{n=1}^{N} C_n \left(r/R\right)^{n+\ell-1} e^{-(r/R)^{\beta}} \mathcal{Y}_{j\ell}^{m_j}(\Omega),$$

with $\beta = 1,2$. The coefficients C_n are determined by the variational technique of minimizing

$$E = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle}$$

with respect to the C_n's. This results in a linear eigenvalue equation and is equivalent to solving the Schrödinger equation.

The resulting radial wave functions are orthogonal and the eigenvalues λ_n are upper bounds on the true energies E_n for every n, i.e. $E_n \leq \lambda_n$, n = 1,…, N. In practice, for N ≥ 10, the lowest 3-4 eigenvalues are stable. This can be seen in a comparison with charmonium results for the Cornell potential,

	ELQ	Variational
A (GeV^2)	0.177	0.179
α_S	0.457	0.448
m_C (GeV)	1.84	1.92
$\langle 1s \rangle$ (MeV)	3067	3067
$\langle 1p \rangle$ (MeV)	3526	3526
$\langle 2s \rangle$ (MeV)	3678	3678
$\langle 1d \rangle$ (MeV)	3815	3814
$\langle 2p \rangle$ (MeV)	3968	3966
$\langle 1f \rangle$ (MeV)	4054	4052

3. Results for a semi-relativistic model.

In what follows, we have included the kinetic energy corrections by using a Hamiltonian of the form

$$H = 2\sqrt{\vec{p}^2 + m^2} + Ar - \frac{4\alpha_S}{3r}\left[1 - \frac{3\alpha_S}{2\pi} + \frac{\alpha_S}{6\pi}(33 - 2n_f)[\ln(\mu r) + \gamma_E]\right] + V_L + V_S.$$

V_L contains the scalar and vector order v^2/c^2 corrections to Ar and V_S includes all v^2/c^2 and one-loop QCD corrections to the short distance potential. Two versions of the model are examined

- $V_L + V_S$ treated as a perturbation
- All terms treated nonperturbatively

Charmonium and Upsilon Parameters and Leptonic Widths

	$c\bar{c}$ Pert	$c\bar{c}$ Non-pert	$b\bar{b}$ Pert	$b\bar{b}$ Non-pert
A (GeV^2)	0.168	0.175	0.170	0.186
α_S	0.331	0.361	0.297	0.299
m_q (GeV)	1.41	1.49	5.14	6.33
μ (GeV)	2.32	1.07	4.79	3.61
f_V	0.00	0.18	0.00	0.09

$\Gamma_{e\bar{e}}$(keV)	Pert	Non-pert	Expt
J/ψ	4.7	1.1	5.40 ± 0.17
$\psi(2S)$	2.8	0.66	2.12 ± 0.12
$\psi(3S)$	2.1	0.54	0.75 ± 0.15
$\Upsilon(1S)$	1.25	1.18	1.31 ± 0.19
$\Upsilon(2S)$	0.58	0.56	0.59 ± 0.03
$\Upsilon(3S)$	0.46	0.44	0.48 ± 0.08

3a. Results for charmonium

In both cases for charmonium, all n = 1, 2 and 3 S, P, and D levels were calculated. The potential parameters and the quark mass were determined by fitting the 1^1S_0, 1^3S_1, 1^3P_J, 2^1S_0, 2^3S_1, 1^3D_1, 3^3S_1, and 2^3D_1 levels to the data. The results are:

	Pert	Non-pert	Expt		Pert	Non-pert	Expt
η_c	2985	2981	2979.7 ± 1.5	η_c'	3599	3624	(3637.7 ± 4.4)
J/ψ	3096.9	3096.9	3096.87 ± 0.04	ψ'	3686	3686	3686.0 ± 0.1
χ_0	3418.4	3415.8	3415.1 ± 0.8	χ_0'	3849	3872	
χ_1	3510.2	3510.4	3510.51 ± 0.12	χ_1'	3946	3951	
χ_2	3556.5	3556.3	3556.18 ± 0.17	χ_2'	3999	3996	
h_c	3527	3524	(3526.21 ± 0.25)	h_c'	3966	3966	
1^3D_1	3809	3790	3770 ± 2.5	2^3D_1	4174	4157	4160 ± 20
1^3D_2	3827	3826	3872 ± 1.0	2^3D_2	4198	4201	
1^3D_3	3831	3845		2^3D_3	4209	4223	
1^1D_2	3824	3825	3836 ± 13.0	2^1D_2	4199	4202	

The resulting M_1 and E_1 widths are

$\Gamma_\gamma(E1)$ (keV)	Pert	Nper	EXP	$\Gamma_\gamma(E1)$ (keV)	Pert	Nper	EXP
$1\chi_{c0} \to \gamma\, J/\psi$	158	169	119 ± 17	$1^3D_2(3826) \to \gamma\, 1\chi_{c1}$	338	314	
$1\chi_{c1} \to \gamma\, J/\psi$	315	357	288 ± 51	$1^3D_2(3826) \to \gamma\, 1\chi_{c2}$	72.3	76.3	
$1\chi_{c2} \to \gamma\, J/\psi$	420	468	426 ± 48	$1^3D_2(3872) \to \gamma\, 1\chi_{c1}$	489	459	
$1h_c \to \gamma\, \eta_c(1S)$	646	670		$1^3D_2(3872) \to \gamma\, 1\chi_{c2}$	111	119	
$\psi(2S) \to \gamma\, 1\chi_{c0}$	45	22	24.2 ± 2.5	$\psi(3770) \to \gamma\, 1\chi_{c0}$	443	291	320 ± 100
$\psi(2S) \to \gamma\, 1\chi_{c1}$	41	33	23.6 ± 2.7	$\psi(3770) \to \gamma\, 1\chi_{c1}$	158	125	280 ± 100
$\psi(2S) \to \gamma\, 1\chi_{c2}$	28	29	24.2 ± 2.5	$\psi(3770) \to \gamma\, 1\chi_{c2}$	6.5	5.6	≤ 330
$\eta_c(2S) \to \gamma\, 1h_c$	9.0	22					

$\Gamma_\gamma(M1)$ (keV)	TH	EX
$J/\psi \to \gamma\eta_c$	2.78	1.2 ± 0.3
$\psi' \to \gamma\eta_c'$	0.45	
$\psi' \to \gamma\eta_c$	0.63	0.8 ± 0.2
$\eta_c' \to \gamma J/\psi$	1.04	
${}^3D_2(3872) \to \gamma\, {}^1D_2$	0.20	

3a. Results for $b\bar{b}$

Again, for both upsilon cases, all n = 1, 2 and 3 S, P, and D levels were calculated. The potential parameters and the quark mass were determined by fitting the 1^3S_1, 1^3P_J, 1^3D_1, 2^3S_1, 2^3P_J, and 3^3S_1 levels to the data. Recalling the parameters:

	$c\bar{c}$ Pert	$c\bar{c}$ Non-pert	$b\bar{b}$ Pert	$b\bar{b}$ Non-pert
A (GeV^2)	0.168	0.175	0.170	0.186
α_S	0.331	0.361	0.297	0.299
m_q (GeV)	1.41	1.49	5.14	6.33
μ (GeV)	2.32	1.07	4.79	3.61
f_V	0.00	0.18	0.00	0.09

The energy level results are:

Upsilon system energy levels

	Pert	Non-pert	Expt		Pert	Non-pert	Expt
$\eta_b(1S)$	9411.6	9416.6	9300 ± 28	$\eta_b(3S)$	10339.5	10342.4	
$\Upsilon(1S)$	9459.5	9459.6	9460.3 ± 0.26	$\Upsilon(3S)$	10359.54	10359.9	10355.2 ± 0.5
$1\chi_{b0}$	9862.5	9862.0	9859.44 ± 0.52	$3\chi_{b0}$	10511.6	10512.1	
$1\chi_{b1}$	9893.2	9895.2	9892.78 ± 0.40	$3\chi_{b1}$	10534.5	10536.8	
$1\chi_{b2}$	9914.0	9911.6	9912.21 ± 0.17	$3\chi_{b2}$	10549.8	10548.1	
$1h_b$	9902.1	9902.3		$3h_b$	10540.9	10541.8	
$\eta_b(2S)$	9996.5	9999.4		1^3D_1	10149.8.	10150.2	
$\Upsilon(2S)$	10020.9	10021.3	10023.26 ± 0.31	1^3D_2	10157.6	10157.4	10161.1 ± 1.7
$2\chi_{b0}$	10228.9	10228.7	10232.5 ± 0.6	1^3D_3	10163.5	10163.0	
$2\chi_{b1}$	10254.0	10256.2	10255.46 ± 0.55	1^1D_2	10158.9	10158.5	
$2\chi_{b2}$	10270.8	10269.0	10268.65 ± 0.55				
$2h_b$	10261.1	10261.8					

Upsilon system E_1 widths

$\Gamma_\gamma(E1)$ (keV)	Pert	Nper	EXP	$\Gamma_\gamma(E1)$ (keV)	Pert	Nper	EXP
$1\chi_{b0} \to \gamma\,\Upsilon(1S)$	23.4	21.1		$\Upsilon(3S) \to \gamma\,2\chi_{b0}$	1.64	1.03	1.30 ± 0.20
$1\chi_{b1} \to \gamma\,\Upsilon(1S)$	29.0	25.9		$\Upsilon(3S) \to \gamma\,2\chi_{b1}$	2.61	1.91	2.78 ± 0.43
$1\chi_{b2} \to \gamma\,\Upsilon(1S)$	33.2	28.2		$\Upsilon(3S) \to \gamma\,2\chi_{b2}$	2.59	2.35	2.89 ± 0.50
$1h_b \to \gamma\,\eta_b(1S)$	41.3	4.85		$\Upsilon(3S) \to \gamma\,1\chi_{b0}$	0.036	0.030	0.0663 ± 0.025
$\Upsilon(2S) \to \gamma\,1\chi_{b0}$	1.12	0.71	1.16 ± 0.15	$\Upsilon(3S) \to \gamma\,1\chi_{b1}$	0.089	0.0029	
$\Upsilon(2S) \to \gamma\,1\chi_{b1}$	1.79	1.32	2.11 ± 0.20	$\Upsilon(3S) \to \gamma\,1\chi_{b2}$	0.13	0.10	
$\Upsilon(2S) \to \gamma\,1\chi_{b2}$	1.76	1.61	2.19 ± 0.20	$\Upsilon(1^3D_1) \to \gamma\,1\chi_{b0}$	18.6	13.1	
$\eta_c(2S) \to \gamma\,1h_c$	2.18	19.8		$\Upsilon(1^3D_1) \to \gamma\,1\chi_{b1}$	10.0	7.92	
$2\chi_{b0} \to \gamma\,\Upsilon(1S)$	6.93	1.77		$\Upsilon(1^3D_1) \to \gamma\,1\chi_{b2}$	0.52	0.46	
$2\chi_{b1} \to \gamma\,\Upsilon(1S)$	7.58	5.02		$\Upsilon(1^3D_2) \to \gamma\,1\chi_{b1}$	19.7	15.4	
$2\chi_{b2} \to \gamma\,\Upsilon(1S)$	8.03	7.15		$\Upsilon(1^3D_2) \to \gamma\,1\chi_{b2}$	5.16	4.54	
$2\chi_{b0} \to \gamma\,\Upsilon(2S)$	10.3	10.5		$\Upsilon(1^3D_3) \to \gamma\,1\chi_{b2}$	22.1	19.3	
$2\chi_{b1} \to \gamma\,\Upsilon(2S)$	14.4	13.3		$2\chi_{b1} \to \gamma\,\Upsilon(1^3D_2)$	1.47	1.58	
$2\chi_{b2} \to \gamma\,\Upsilon(2S)$	17.6	14.3		$2\chi_{b2} \to \gamma\,\Upsilon(1^3D_2)$	0.47	0.43	

4. Conclusions and Outlook

- The semi-relativistic model provides a quantitatively good description of the charmonium and upsilon spectra. Only the $^3D_1(3770)$ charmonium state is poorly described, probably because S-D mixing is not included.

- The Lorentz structure of the confining potential is interesting. In both cases ($c\bar{c}$ and bb) the perturbative treatment of the spin-dependent interactions always favors a pure scalar confining potential, while treating the spin terms non-perturbatively favors a scalar-vector mixture ~18% vector for $c\bar{c}$, ~10% vector for $b\bar{b}$.

- The calculated E_1 decays compare favorably with experiment. Transitions between $J/\psi,\chi$ and ψ' appear to be dominated by spin rather than open channel effects.

- Based on the model considered here, the X(3872) cannot be explained solely in terms of a charmonium $^3D_2$ state described by a potential. Spin effects alone can only separate the $^3D_2$ from the $^3D_1$ by 40 MeV or so, which suggests that the inclusion of open channel effects is essential if this identification is to be established.

- The X(3943) is compatible with a 2P charmonium state.

- For reasons that are not completely clear, the $b\bar{b}$ system seems to be better described with the perturbative treatment.

- The potential for unequal mass systems has also been calculated and can be used to investigate the D_S , B_S , and B_C mesons (Gupta, SR & WWR, 1981, 1985).